Applications of QFT robust control techniques to marine systems

R. Muñoz-Mansilla, J. Aranda, J.M. Díaz. D. Chaos, A.J. Reinoso

Abstract— **In this work an overview of the application of the Quantitative Feedback Theory (QFT) technique to different marine systems is presented. Namely the problems of the stabilization of a high-speed craft, the dynamic positioning of a moored platform and the tracking control of a hovercraft are studied. An interesting question is that the plants have less degree of freedom for actuation and is more difficult to control. The three multivariable nonlinear problems are tackled by different procedures. Performance evaluation analyses and simulations in different conditions are carried out. It is shown that robust techniques based on QFT methodology result feasible and very suitable; and therefore they constitute attractive alternatives in the application of stabilization, dynamic positioning and tracking control of advanced marine systems.**

I. Introduction

RECENTLY, an advanced marine system has been required to improve maneuverability of systems. For marine systems which can afford surpassing maneuverability under severe environmental conditions, it is of great importance to develop an advanced control system. Marine systems are defined as the systems which operate marine crafts and equipment to meet maneuvering requests. As for marine crafts, there are so many types of crafts on and under the sea. Ships, offshore platforms, and underwater vehicles, are typical examples of marine crafts.

Development of control logic is the key technology to improve the quality of marine systems. Then, it is important to design advanced practical control algorithms of marine systems.

According to [1], a design method based on modeling, such as robust control, is more effective for the design of control systems with several control characteristics, if the vehicle mathematical model is estimated to some extent. It is of great importance for designing marine control systems to have robustness against environmental disturbances, tracking response to reference command, prevention of coupling motion, and faster response. Good robustness, accurate tracking response, and prevention of coupling motion are the main problems. Therefore, the robust control logic is presented as a good alternative to apply to these kinds of problems.

R. Muñoz-Mansilla is with the Automatic Control and Computer Engineering Department, National Distance University of Spain, Madrid, 28040, Spain (corresponding author to provide phone: 00 34 913988254; fax: 00 34 913987690; e-mail: rmunoz@dia.uned.es).

J.Aranda, J.M. Díaz, and D. Chaos are with the Automatic Control and Computer Engineering Department, National Distance University of Spain, Madrid, 28040, Spain, (e-mail: jaranda@dia.uned.es, josema@dia.uned.es, dchaos@dia.uned.es).

A.J. Reinoso, is with the GSyC/Libresoft Department, Rey Juan Carlos University, Madrid, 28933, Spain, (e-mail: ajreinoso@gsyc.urjc.es)

Numerous robust control algorithms for controlling marine systems have been examined, such as LQG optimal control, H_∞ robust control, μ-synthesis, or neural networks, for the control problem of roll stabilization [2], tracking [3], ship maneuvering [4], and ship positioning [5, 6] and course-keeping [7, 8].

Another question to take into account is the fact that marine systems are normally underactuated vehicles, i.e., systems with fewer independent control inputs than degrees of freedom to be controlled. These systems are usually costly and impractical to fully actuate. In addition, they are not fully feedback linearizable and exhibit non-holonomic restraints [9, 10]; therefore classical nonlinear techniques are not applicable and new designs should be examined.

Taking into account all mentioned above, currently control problems of underactuated marine vehicles with requirements of decoupling motions and robust performance against disturbance and uncertainties in the plant model become very attractive and motivate the development of alternative control design techniques.

With this purpose, in this work we present and analyze the application of diverse strategies based on a robust control methodology, concretely Quantitative Feedback Theory (QFT) to different marine systems, which has not been very common in naval systems. Especially authors present in this work the stabilization of a high-speed craft, dynamic positioning of a moored floating platform, and tracking control of an hovercraft.

QFT [11, 12, 13, 14] is a frequency domain robust-design methodology for control systems where the plant is uncertain and/or there are disturbances acting on the plant. The idea has been applied to scalar, multivariable, linear, nonlinear and invariant/time varying uncertain systems. The technique has attracted considerable interest in theory and engineering applications, such as aeronautics, aerospace industry, robotics, electronics and electrical engineering. Samples of application are interferometers, optical disk drivers, electro-hydraulic [15], pneumatic actuators [16] and flight control [17]. Initially it has not been very common in marine systems.

Consequently, the present work has as objective to verify that QFT synthesis is a good alternative for the control problem of stabilization, dynamic positioning, and position tracking of different types of advanced marine vehicles.

The paper is organized as follows. In section II an introduction of QFT methodology is presented. In section III the model and control solution for the fast ferry stabilization problem is developed. Section IV presents the model and the

Preprint submitted to The 9th IEEE International Conference on Control & Automation. Received May 7, 2011.

problem of dynamic positioning of a floating platform. Section V shows the results above the hovercraft. Section VI encapsulates the conclusions.

II. FORMULATION OF THE QFT DESIGN

QFT is a frequency domain design methodology that was introduced by Horowitz [11]. The foundation of QFT is the fact that feedback is primarily needed when the plant is uncertain and/or there are disturbances acting on the plant. Hence, for the sample of the high-speed craft, the model parameters identified for angle of incidence different μ from the nominal case with μ=135º are considered as uncertainties. Next, the platform and hovercraft present uncertainties in the model parameter. The three cases have output disturbances due to the seaway. Therefore, at first sight, the feedback controls of the three samples of this work seem to be a good example for using the QFT technique.

Horowitz published works for SISO plants [18], for linear MIMO [19, 20], and for various classes of nonlinear time varying systems [21].

The QFT design procedure involves four basic steps: generation of plant templates, computation of QFT bounds, design of the controller (loop shaping), and analysis of the design.

The plant templates are defined as the plant frequency response set at a fixed frequency. Given the plant templates, QFT converts closed-loop magnitude specifications into magnitude constraints on a nominal open-loop function (QFT bounds). A nominal open-loop function is then designed to satisfy simultaneously the plant template constraints as well as to achieve nominal closed-loop stability (loop shaping). It is defined the open loop function $L(j\omega)$ as the product of the controller transfer function and the plant transfer function. In any QFT design, it is necessary to select a frequency array for computing templates and bounds. In the three cases of study, the range of frequencies chosen belongs to the seaway spectrum, with natural frequency $\omega \in [0.39, 3]$ rad/s.

Since the transfer function models are considered as a nominal plant with an uncertain set, a robust performance problem is presented, because the performance specifications must be satisfied for all the possible systems admitted by the specific uncertainty model.

To begin with, the formulation of what is the required behavior of the closed-loop system is necessary. The specifications must be given in terms of frequency response. Therefore, it is necessary to translate temporal constraints into frequency domain specifications, and normally it is not a trivial task.

QFT closed-loop specifications used are the gain and phase margin stability (1) and the output disturbance rejection or sensitivity reduction (2):

$$\left|\frac{P(j\omega)\cdot G(j\omega)}{1+P(j\omega)\cdot G(j\omega)}\right| \le \lambda(\omega); \forall\, \omega > 0, \forall\, \mathrm{P} \in \wp \tag{1}$$

$$\left|\frac{1}{1+P(j\omega)\cdot G(j\omega)}\right| \le \delta_s; \forall\, \omega > 0, \forall\, \mathrm{P} \in \wp \tag{2}$$

After the stability and performance bounds have been computed, the next step in a QFT design involves the design (loop shaping) of a nominal function that meets the design bounds [22]. The nominal loop $L(j\omega)$ has to satisfy the worst case (intersection) of all bounds.

Once the controller parameters are designed by using QFT design, the system in closed-loop dynamic is simulated in order to prove if the control meets the specifications.

III. STABILIZATION CONTROL OF A HIGH-SPEED CRAFT

The main objectives in the design and construction of high-speed crafts are passenger comfort and vehicle safety. The vertical accelerations associated with roll, pitch and heave motions are the principal cause of motion sickness.

Accordingly, the purpose of this work is the design of a stabilization control for a high speed craft and the study focuses on the coupled dynamics of heave, pitch and roll motions. The analyzed couplings are derived from the possible interference effects of the appendages on the ship's hull motions when the angle of incidence differs from 180º. The active stabilization surfaces employed are one T-Foil on bow, two flaps on stern, and two lateral fins. Thus, one of the problems observed is the fact that the actuator action to control the roll mode generates a component in the pitch mode. Similarly, the T-Foil and flaps surfaces produce, together with the control action itself, a coupling with the roll mode.

Under assumption of small coupling effects due to the actuators, two independent control designs for each one of the two dynamics works successfully [23]. However, when cross-coupling effects are notable, new approximations must be tried in order to reduce the longitudinal and transversal dynamics in the multivariable high speed craft system.

This work proposes an approach to handle multi-input multi-output (MIMO) robust control problems of stabilization of marine vehicles with coupled dynamics by using a combination of two control techniques. In particular, the procedure consists of blending the QFT technique with the Eigenstructure Assignment (EA) technique. A good decoupling performance can be achieved for a nominal plant model with an adequate EA design, which can provide transformation of the MIMO system into decoupled SISO subsystems. Then the SISO QFT technique is used to achieve robust performance under plant uncertainties.

A. Model of the high-speed craft

Prior to stabilization control, mathematical models of the heave, pitch and roll dynamics are built by using system identification methods for the cases of angle of incidence between 90º and 180º [24]. The method identifies linear continuous models and uses data obtained via experiments with a scaled-down replica 1:25 of the fast ferry (Fig. 1) in CEHIPAR (El Pardo Model Basin, Spain) and a sea behaviour program named PRECAL, which reproduces specified conditions (different types of waves, ship speeds, and angle of incidence), and uses a geometrical model of the high speed craft to predict its dynamic behavior [25]. Furthermore, dynamics of T-Foil, fins and flaps are modelled.

The model of the system has three outputs: the vertical acceleration in heave (*acvh*), the vertical acceleration in pitch (*acvp*), and the roll angular velocity (ω_{roll}). The control inputs are: the angle of attack of the flaps (α_P), T-Foil (α_H), and the lateral fins (α_R).

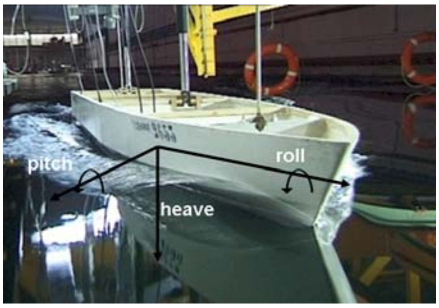

Fig. 1. Model replica of the fast ferry.

Thus, a single degree of freedom (DOF) 3x3 MIMO system is presented, with three inputs and three outputs. Fig. 2 shows the block system diagram with the three modes, where the coupling of the modes is considered as a consequence of the control surface action in different incidence angles of the seaway.

The stabilization problem is stated as a robust control design of a coupled system, with a nominal plant (the models identified for angle of incidence μ=135º) with uncertainties (the ship models responses for the rest of angles of incidence), and the seaway as the input disturbances.

Since heave, pitch and roll have restoration forces, the almost lack of inherent motion damping means that small additions to this damping can produce large reductions in the response. So, the best way of reducing it is to increase damping by using the active stabilization devices.

Consequently, the controllers must be set up in such a way as to ensure that the actuators develop moments and forces which oppose the moments and forces provided by the waves. For the particular high speed craft system, the following control objectives are required: i) system stability; ii) heave, pitch, roll reduction; iii) no saturation on T-Foil, flaps and fins ($\left|\alpha\right|_i < \pm 15^{\circ}; \forall i$).

B Combined EA/QFT control design.

The MIMO problem is firstly handled with EA technique in order to decouple the dynamics, which results in three SISO systems to solve with QFT design.

1) EA design: The EA [26] is a multivariable control design technique based on the fact that through an adequate assignment of the closed-loop eigenvalues and eigenvectors, improved transient dynamics and their coupling dynamics can be achieved. This technique has been widely applied to the design of flight control systems [27, 28].

EA design can only be done for a specific system model, so the results will be focused on the nominal plant, corresponding to the case with angle of incidence μ=135º. The basic principle is to assign the eigenvalues and eigenvectors of a closed-loop control system to their desired values through state or output feedback strategies. For a specific state, if some elements in its corresponding eigenvector can be assigned to zero, through appropriate system design, decoupled transient responses between this state and the other related states can be achieved.

The EA design requires the system model in state-space equations, so the system 3x3 transfer functions matrix are translated into the form

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\cdot\boldsymbol{x} + \boldsymbol{B}\cdot\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\cdot\boldsymbol{x} \end{aligned} \tag{4}$$

where the control input vector is $\boldsymbol{u}=[\alpha_{R,}\ \alpha_{P,}\ \alpha_H]^{\mathrm{T}}$ and the output vector is $\boldsymbol{y}=[\ \omega_{roll,}\ ,acvp,\ acvh]^{\mathrm{T}}$.

A detailed study about the algorithms for obtaining the gain $\boldsymbol{K}$ matrix such that the eigenvalues and eigenvectors of the closed-loop system matrix $\boldsymbol{A}$-$\boldsymbol{BKC}$, obtained when using the output feedback control equation $\boldsymbol{u} = -\boldsymbol{K}\cdot\boldsymbol{y}$, achieve the required decoupled behavior can be found in [26, 29, 30].

Finally, by using these eigenstructure assignment algorithms, the feedback gain control $\boldsymbol{K}$ matrix is obtained as

$$\mathbf{K} = \begin{pmatrix} 0.047 & -8.6526 & 1.3301 \\ -0.0037 & -3.6554 & 0.5620 \\ 0.0947 & -9.1661 & 1.3670 \end{pmatrix} \tag{5}$$

With $\boldsymbol{K}$ matrix, it is shown that the coupling effects on roll, heave and pitch modes due to actuators have become weaker.

But as above-mentioned, the EA design has been only done for the specific nominal case with angle of incidence μ= 135º, therefore the coupling effects can be still quite strong for other angles of incidence. In order to achieve robust performance QFT design is proposed.

2) QFT design for the EA design results: From the EA design results, QFT design faces the closed loop system described by Fig. 2.

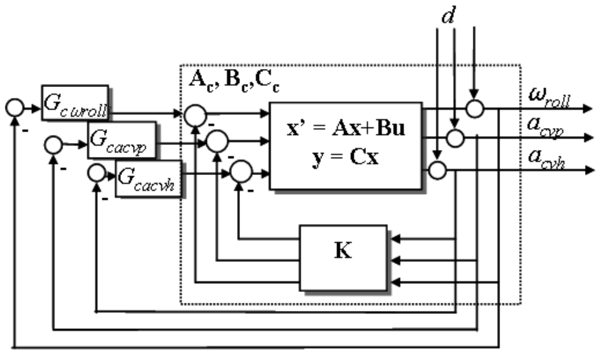

Fig 2. Ship system with combined EA/QFT.

The state-spaces matrices are the following:

$$\boldsymbol{A_c} = A - BKC; \boldsymbol{B}c = B; C_c = C \tag{6}$$

For QFT design purposes, the plant model in state-space equations is transformed into system transfer function. Thus, the corresponding closed-loop transfer function $\boldsymbol{P}$ matrix is:

$$\boldsymbol{P} = \boldsymbol{C_c}\left(\boldsymbol{sI} - \boldsymbol{A_c}\right)^{-1}\boldsymbol{B}_c = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix} \quad (7)$$

As seen, EA design provides decoupled responses of roll, pitch, and heave dynamics for the case with angle incidence μ = 135º, thus the cross coupling transfer functions can be considered as approximately zero, i.e., Pij (i≠j)=0. Therefore the result consists of three independent SISO systems (P_{11}, P_{22}, P_{33}) to control with QFT design, in order to perform robustness against the parameter uncertainties over the angle of incidence state and output disturbances.

3) QFT design for the roll dynamic subsystem: The specifications for robust stability (λ_{11}= 1.2) and performance bounds (δ_{s11}=1.4) fixed for the QFT design guarantee adequate gain margins and output disturbance rejection.

The stability margins are arbitrary chosen with the value λ_{11}= 1.2, and implies a robust stability with at least 1+(1/λ_{11})=1.71 lower gain margin, and 180º-arcos(0.5/ λ_{11}^2-1)=49.25º lower phase margin. This makes the following inequality satisfies for all frequencies:

$$\left|\frac{P_{11}G_{c\omega roll}}{1+P_{11}G_{c\omega roll}}\right| \le \lambda_{11} = 1.2, \quad \omega \ge 0 \quad (8)$$

The bounds at low frequencies (ω ≤ 3 rd/s) are calculated in order to satisfy the inequality for disturbance rejections:

$$\left|\frac{y}{d}\right| = \left|\frac{\omega_{roll}}{d}\right| = \left|\frac{1}{1+P_{11}G_{c\omega roll}}\right| \le \delta_{s11} = 1.4; \quad \omega \le 3\,rad/s \quad (9)$$

The control $G_{c\omega roll}$(s) must be designed such that open loop function $L_{\omega roll}(j\omega)$:

$$L_{\omega roll}(j\omega) = G_{c\,\omega roll}(j\omega)\cdot P_{11}(j\omega) \quad (10)$$

satisfies the worst case of all bounds (intersection). The controller designed is a first order filter:

$$G_{c\omega roll}(s) = 1.5\cdot\frac{\left(\frac{1}{0.58}s+1\right)}{\left(\frac{1}{9.77}s+1\right)} \quad (11)$$

The same procedures are carried out for the design of G_{cAcvp} and G_{cAcvh}, where finally the controllers designed are:

$$G_{acvp}(s) = 3.1\cdot\frac{\left(\frac{1}{0.026}s+1\right)}{\left(\frac{1}{0.035}s+1\right)} \quad (12)$$

$$G_{acvh}(s) = 5.4\cdot\frac{\left(\frac{1}{2.42^2}s^2+\frac{0.14}{2.42}s+1\right)}{\left(\frac{1}{2.91^2}s^2+\frac{0.1}{2.91}s+1\right)\left(\frac{1}{3.56}s+1\right)} \quad (13)$$

Combining the EA and QFT design results, the whole MIMO system is obtained, as depicted in Fig. 2. Simulations of the MIMO control system are undertaken to evaluate the whole system performance.

Specifically, simulations using 40 knots ship speed, regular waves with 0.8 meters of amplitude frequency [0.39,3] rad/s, and irregular waves with sea state SSN= 4, 5, 6 were employed, and the percentage reduction is measured. To illustrate, Table I shows the value of percentage reduction in roll, pitch and heave responses with combined QFT-EA control in the whole system for the case of ship speed V=40 knots and waves in sea state SSN= 4 and 5.

TABLE I

REDUCTION PERCENTAGE ROLL, HEAVE, PITCH WITH QFT-EA CONTROL. V = 40 KNOTS. SSN=4 AND 5

‘	μ (º)	Reduction ω_{roll} (%)	Reduction *acvp* (%)	Reduction *acvh* (%)
SSN4	105	0,8%	34,5%	2,8%
	120	12,1%	44,8%	4,7%
	135	178,4%	48,7%	2,6%
	150	45,8%	44,4%	3,9%
	165	86,9%	0,6%	4,1%
	180	65,6%	13,1%	1,9%
SSN5	105	1,1%	34,3%	3,3%
	120	40,4%	45,7%	6,0%
	135	87,2%	49,4%	5,0%
	150	28,7%	46,7%	7,1%
	165	12,4%	46,7%	7,1%
	180	50,8%	14,4%	0,01%

In conclusion, in the first example of the high-speed craft, the MIMO problem is firstly handled with EA technique to decouple the three dynamics. EA provides a straightforward physical understanding of the design process and gives designer flexibility in system performance. In order to achieve robust performance for other incidence angles and state seas, QFT design is proposed. Performance evaluation analyses and simulations of temporal responses with different waves and incidence angle are carried out. It is demonstrate that one fixed control reaches the desired closed-loop specifications for different conditions and the designed closed-loop system achieve decoupling between the three dynamics and damped responses. It is shown that the combined EA/QFT technique is a robust method very suitable for the implementation, and that accomplishes the objectives efficiently. We have verified that this method is an attractive alternative to handle MIMO coupled systems.

IV. DYNAMIC POSITIONING PROBLEM OF A MOORED PLATFORM.

In this second part of the work, the marine system studied consists of a moored floating platform. Position mooring and dynamic positioning are required in many offshore oil and gas field operations, such as drilling, pipe-laying, tanking between ships, and diving support [31]. Therefore, these platforms require a high level of precision in the positioning for optimal operations. In addition, they are subject to environmental charges combined of waves, wind and currents, which affect such the stability as the positioning.

Consequently, robustness to plan uncertainties as well as rejection to environmental disturbances are important features of a dynamic positioning system. In addition, the system has lees degree of freedom for actuation, thus a control problem of an underactuated system is raised.

A moored floating platform is a nonlinear system, but for this study a linear approximation in the state-space form is considered in order to design the controller. Hence, the model of the moored platform is a single degree of freedom single-input/multi-output (SIMO) linear time invariant system. The goal is to minimize the drift and angular deviation resulting from the wave action by appropriate thrusters control.

The control design process to handle the SIMO system is based on the transformation of the problem into the design of two sequential SISO systems [11]. Thus, it is solved by an iterative multi-stage sequential procedure.

Taking into account all above, the challenge is to study the effectiveness of the QFT technique to accomplish the dynamic positioning of an underactuated system.

A. Model of the moored platform

The system consists of a floating platform model [32] that is anchored to the bottom of the ocean and equipped with two thrusters, as it is showed in Fig. 3. The objective is to achieve an appropriate thrusters control in order to minimize the drift Y and angular deviation ϕ resulting from the wave action.

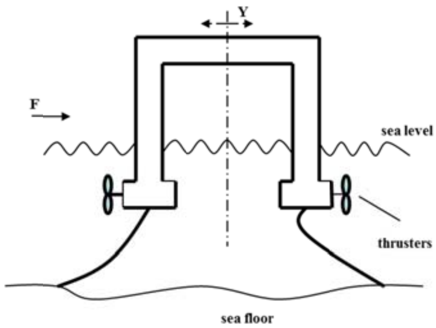

Fig 3. Moored floating platform.

The model of the system has two outputs $\boldsymbol{y}$ (the horizontal drift Y and angular deviation from the vertical axis ϕ), one control input u (the force delivered by the thrusters F_u) and two disturbance inputs $\boldsymbol{d}$ (the force F and the torque M from the wave action). Therefore a single degree of freedom (DOF) SIMO system is presented, with one single input F_u and two outputs (Y, ϕ).

For design purposes, the system transfer function can be described as:

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{P_{plant}}(s) \cdot u + \boldsymbol{P_d}(s) \cdot \boldsymbol{d} \\ u &= -\boldsymbol{G_{control}}(s) \cdot \boldsymbol{y} \end{aligned} \quad (14)$$

where $\boldsymbol{y} = [Y, \phi]^T$ is the output plant, $\boldsymbol{P_{plant}}(s)$ is a transfer functions matrix (2x1) that connects the input u with the output $\boldsymbol{y}$, and $\boldsymbol{P_d}(s)$ is a transfer functions matrix (2x2) that connects the disturbance $\boldsymbol{d}$ with the output $\boldsymbol{y}$. The control structure is schematically displayed in Fig. 4.

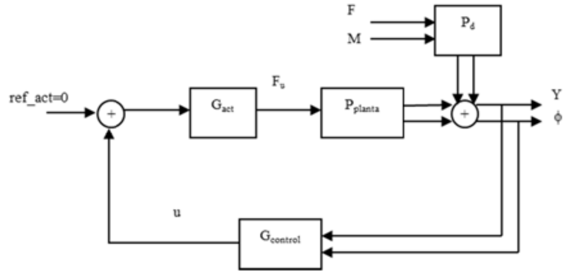

Fig.4. Single DOF SIMO system with disturbances at the plant's output.

In these conditions, the problem of interest is how to design the controller $\boldsymbol{G_{control}}$. For the particular moored floating platform system, the following control objectives ([39]) are required: *i)* Reduce the drifting action by using the actuators control; *ii)* Maintain the horizontal drift $|Y| < 0.025$ m. and the angular deviation $|\phi| < 3$ degrees, *iii)* Keep the force delivered by the thrusters $|F_u| < 0.25$ N.

B A multivariable QFT controller

The specifications must be given in terms of frequency response. For the particular case of the design of the dynamic positioning system for the moored platform model, the specifications ($|Y| < 0.025$ m, $|\phi| < 3$ deg.) are given in temporal domain. Therefore, it is necessary to translate these constraints into frequency domain specifications. The QFT specifications used are: the gain and phase margins stability (1) and the output disturbance rejection (2).

The control law of the system in Fig. 4 is:

$$\boldsymbol{G_{control}}(s) = \begin{pmatrix} k_1(s) & k_2(s) \end{pmatrix} \quad (15)$$

Solving (14) and (15), it yields one equation with two unknown quantities, k_1 and k_2:

$$\begin{aligned} &(\hat{p}_{13} + k_1)Y + (\hat{p}_{23} + k_2)\phi = (\hat{p}_{13}p_{11} + \hat{p}_{23}p_{21})F + \\ &+ (\hat{p}_{13}p_{12} + \hat{p}_{23}p_{22})M \end{aligned} \quad (16)$$

The control design process is based on this equation, which aids in transforming the problem into the design of two sequential SISO systems. Thus, it is solved by an iterative multi-stage sequential procedure, in such a way that the solution of k_1 in the first system is used in the design of k_2 in the second system, and vice versa. The stages repeat successively up to k_1 and k_2 meet the objectives for the SIMO system. Finally, the control design procedure has been completed in five stages, and the designed controllers are:

$$k_2(\mathrm{s}) = \frac{\left(\frac{1}{1.26^2} s^2 + \frac{2 \cdot 0.074}{1.26} s + 1\right)}{\left(\frac{1}{1.8^2} s^2 + \frac{2 \cdot 0.14}{1.8} s + 1\right)} \quad (17)$$

$$k_1(\mathrm{s}) = -0.28 \cdot \frac{\left(\frac{1}{0.52} s + 1\right)}{\left(\frac{1}{1.95} s + 1\right)} \quad (18)$$

Temporal responses of the SIMO system (Fig. 3) in closed-loop dynamic are shown. It is observed the controller achieves the output *Y(t)* gets into the range ±0.025 m before t = 80 seconds (see Fig. 5). Regarding $\phi(t)$, it is observed that it remains the range given by the specification ±0.7º from the beginning. Therefore, it is shown that the control meets the original specifications and therefore, achieves the positioning system.

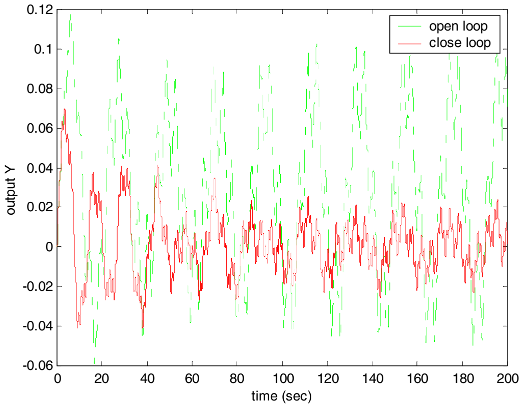

Fig.5 Comparison of temporal response *Y* in open loop (dashed line) and closed loop (solid line).

To sum up this second work, the control problem of the underactuated system is solved by an iterative multi-stage sequential procedure. Simulation results demonstrate that the designed control achieves the positioning system. This, it can be concluded that the fact that robust control techniques based on QFT design are applied successfully to a typical marine control problem, and secondly, from the point of view of theory of control, this case shows again that QFT is a feasibly methodology to solve the problem of rejection to disturbances in an underactuated system of a dynamic positioning problem.

V. TRACKING CONTROL PROBLEM OF HOVERCRAFT

The past few decades have witnessed an increased research effort in the area of trajectory tracking control for underactuated autonomous vehicles. The present work is devoted to solve the problem of tracking control of underactuated vehicles, specifically hovercrafts, which can be seen as a special case of a surface vessel where the essential nonlinearity has been captured.

The system consists of a hovercraft equipped with two longitudinal propellers that provide the thrust to move the vehicle forward (and backward) and to make it turn.

The model is a second order nonlinear system with plant uncertainties and with less degree of freedom for actuation (2x3 MIMO), and therefore is more difficult to control. The goal is to track course and velocity.

As aforementioned, there are many publications related to control problem of underactuated nonlinear vehicles. However, disturbances and uncertainties in the plant are not usually considered in most of the cases, and consequently the designed controllers do not have robust performance. With this purpose, the control problem is tackled as a multivariable nonlinear control design using QFT technique. The approach to non-linear QFT synthesis follows the ideas described in [33], where a local linearization of the nonlinear plant about closed-loop acceptable outputs is proposed.

A. Model of the hovercraft

The nonlinear model for the underactuated hovercraft was obtained from the ship model in [34]. The general kinematic and dynamic equations of motion of the hovercraft can be developed using a global coordinate frame $\{XY\}$ and a body fixed coordinate frame $\{X_BY_B\}$ that are depicted in Fig. 6. Considering that the state vector is the non-linear state equations are [35]:

$$
\begin{aligned}
\dot{x} &= u\cdot\cos\theta - v\cdot\sin\theta \\
\dot{y} &= u\cdot\sin\theta + v\cdot\cos\theta \\
\dot{\theta} &= r \\
\dot{u} &= r\cdot v + \frac{1}{m}\cdot F_x - \frac{1}{m}\cdot r_l\cdot u \\
\dot{v} &= -r\cdot u - \frac{1}{m}\cdot r_l\cdot v \\
\dot{r} &= \frac{1}{J}\cdot T_\theta - \frac{1}{J}\cdot r_r\cdot r
\end{aligned}
\qquad (19)
$$

where *x, y,* θ denote the position and the orientation of the hovercraft in the earth-fixed frame *XY*; *u, v* are respectively the surge and sway velocities in the body-fixed frame X_BY_B, and *r* is the yaw rate. The system has two control inputs: $F_x = (F_s + F_p)$ is the control force in surge, and $T_\theta = l\cdot(F_s - F_p)$ is the control torque in yaw. As seen, we do not have an available control in sway, so there is a non-holonomic constraint. The hovercraft nominal parameters have been computed experimentally in a real system: mass m = 0.894 Kg, moment of inertia J = 0.0125 Kg·m^2, moment arm l = 0.0485 m, friction coefficients r_l = 0.10 Kg/s, and r_r = 0.05 Kgm^{2}s, and $F \in [0.342, -0.121]$ N.

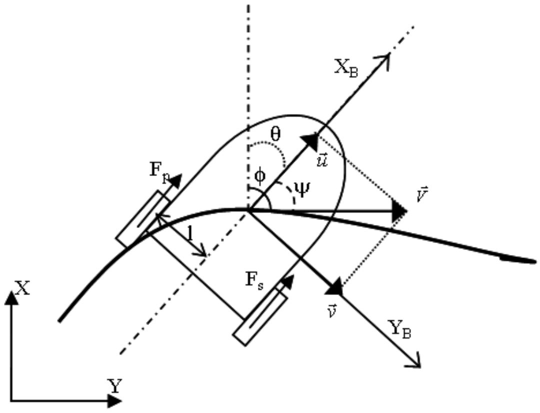

Fig. 6. Model of the hovercraft. Body fixed X_BY_B and earth fixed coordinate frames *XY*.

The control objective is to achieve the tracking control. The two outputs are: the tangential velocity *V,* defined as $V = \sqrt{u^2 + v^2}$, and the derivative of the course angle $\dot{\phi}$, defined as the angle that the tangent of the trajectory in the X_BY_B plane makes with the inertial *X*-axis (see Fig. 6), that is, $\phi = \arctan(\dot{y}/\dot{x})$, also defined from the attitude angle θ and offset angle in yaw ψ as $\phi = \psi + \theta$.

For the statement of the robust control problem of course and velocity, specifications for straight lines or circumferences are given. The tracking specifications assign a peak overshoot M_p=2 and settling time t_s=1.5 s. The system contains plant uncertainties (the dynamic parameters and the friction coefficients). Therefore, a robust technique, in particular QFT, is proposed to do the feedback control.

B. Nonlinear QFT design based on local linearization.

1) Step 1. Local linearization: A non-linear control problem of a 2 degree of freedom (DOF) MIMO 2x2 nonlinear system is raised. The solution proposed is a nonlinear QFT technique [36] based on local linearization [40]. This technique is based on the Schauder fixed point theorem [44] and the idea consists of the replacement of the nonlinear plant N by a set of LTI plants $\{P\}$ (the equivalent linear family ELF) and a set of attached disturbances D, giving as a result a linear multivariable control problem that will be solved in the linear QFT framework.

2) Step 2. Linear MIMO QFT Design: Therefore, once the ELF is computed, a QFT synthesis of a two DOF MIMO system (Fig. 7) is raised. The set of transfer function models $\{P\}$ obtained are interpreted as a set of parametric uncertain systems. Also, an additional parameter uncertainty is considered. A robust performance problem is thus presented. From fixed point theory, the problem is transformed into the design of two sequential 2 degree of freedom MISO (multi-input, single-output) systems [20]. Each MISO problem consists of the design of a controller g_i (i = 1, 2) and prefilter f_{ii} for the plant q_{ii} whose output is V for i=1 and $\dot{\phi}$ for i=2, such that satisfy stability and robust performances and achieve the tracking.

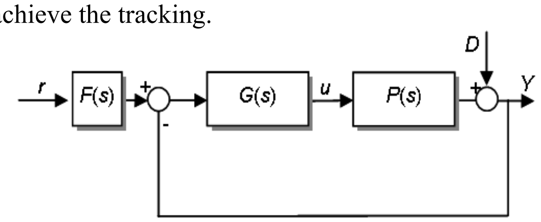

Fig. 7. Two DOF 2x2 MIMO system to solve in the design of G and F.

3) Results: The set of closed-loop acceptable outputs y_a chosen include straight line and circular trajectories, with $V \in [0.1, 1]$ m/s and $\dot{\phi} \in [0, 0.4]$ rd/s. According to these trajectories, local linearization gives the following ELF:

$$\{\wp\} = \left\{ \begin{array}{l} P(s) = \begin{pmatrix} \dfrac{k_1}{s^2+a} & \dfrac{-k_2}{s^3+a\cdot s} \\ \dfrac{-5\cdot s}{s^2+a} & \dfrac{80\cdot s^2+k_3 s+k_4}{s^3+a\cdot s} \end{pmatrix}; \\ a \in [0.01, 0.16]; k_1 \in [0.1, 0.52]; \\ k_2 \in [0.1, 2.4]; k_3 \in [80, 138]; \\ k_4 \in [0.1, 12.8]; \end{array} \right\} \quad (20)$$

The computation of D is not a trivial question, so it can be well approximated [37] by the larger set $D = \{d / |d(t) < D_M|\}$, in order to compute the worst case disturbance effect. For this case, D_M is estimated as $D_M = [5.9, 2.1]^T$.

Next step in QFT design is to compute the bounds. After the closed-loop specifications in terms of frequency response are computed for each MISO subsystem, control g_i is designed via loop shaping such that the open loop function $L_i = g_i \cdot 1/q_{ii}$ (i=1,2) satisfies the worst case of all bounds. Then, f_{ii} is designed to meet tracking specifications. The results obtained are:

$$g_1(s) = 15.32 \frac{1/1.3^2 s^2 + 2*0.7/1.3s + 1}{(1/5.5s+1)(1/6.04s+1)(1/121s+1)}; \quad (21)$$

$$f_{11}(s) = \frac{1}{0.17s+1} \quad (22)$$

$$g_2(s) = 11.2 \frac{(1/2.8s+1)}{(1/0.4s+1)(1/204s+1)} \quad (23)$$

$$f_{22}(s) = \frac{1}{0.31s+1} \quad (24)$$

Finally, the analysis of the whole system in closed-loop dynamic is done. Fig. 8 shows the tracking of the non-linear nominal plant for a circular reference, with a quite good agreement. In addition, robustness is confirmed by Monte Carlo simulations with parameter variability in an uncertain rank of 10% (Fig. 9).

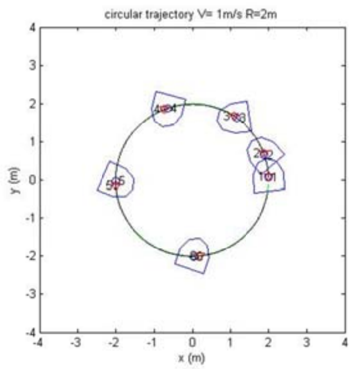

Fig. 8. Circular trajectory for the nominal nonlinear plant with V=1 m/s and $\dot{\phi}$=0.5 rd/s. Reference position for each time marked by triangles.

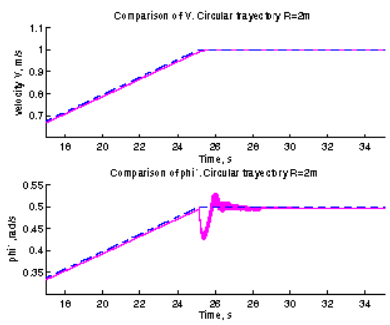

Fig. 9. Robust assessment. Temporal responses of V and $\dot{\phi}$. In dashed line circular reference R=2 m, with V=1 m/s and $\dot{\phi}$=0.5 rd/s.

In conclusion, nonlinear QFT design provides robust performance and accomplishes the objectives efficiently, and results very suitable for the implementation. We have verified that this method is an attractive alternative for robust design of multivariable nonlinear non-holonomic uncertain systems.

Preprint submitted to The 9th IEEE International Conference on Control & Automation. Received May 7, 2011.

VI. CONCLUSIONS

In this work an analysis of the application of the QFT technique to different marine systems is presented. Especially the problems of stabilization of a high-speed craft, dynamic positioning of a moored platform and the tracking control of a hovercraft are studied. The plant models of the three marine vehicles raise different difficulties such as nonlinearities and non-holonomic constraints that must to be taken into account to design the control. In addition, uncertainties in the plant are considered in each case (the plant model with different angles of incidence in the craft system, the plant model with diverse characteristics in the moored platform, and the set of equivalent linear families in the hovercraft). The disturbances considered are the seaway in all the examples. Finally it is shown that multivariable (such nonlinear as linear) QFT design is a robust method very suitable for the implementation, and that accomplishes the objectives efficiently. We have verified that this method is an attractive alternative for robust design of these kinds of marine systems.

ACKNOWLEDGMENT

Authors gratefully appreciate the support given by the Spanish Ministry of Education and Science for its support under grant DPI2009-14552-C02-02.

REFERENCES

[1] I. Yamamoto, “Robust and non-linear control of marine system” *International Journal of Robust and Nonlinear Control*, vol 11, pp. 1285-1341, 2001

[2] J. Van Ameronger, PGM Van der Klugt “Rudder roll stabilization for chips”. *Automatica*. vol. 26, No. 4, pp.679-690, 1990.

[3] K. Djouani, Y. Hamam, “Mimimum time-energy trajectory planning for automatic ship berthing”. *IEEE Journal of Oceanic Eng.,* vol.20, No.1, pp. 4-11, 1995.

[4] J.Y. Juang, B.C Chang, “Robust control theory applied to ship manoeuvring”. *Proc. IEEE Conf. on Decision and Control*, pp. 2186-2191, 1999.

[5] R.S. Burns, “The use of artificial neural networks for the intelligent optimal control of surface shop”. *IEEE Jour. of Oceanic Eng.* vol. 20, No.1, pp.65-72, 1995.

[6] M.R., Katebi, I. Yamamoto, M. Matsuura and M.J. Grimble, “Robust dynamic ship positioning control system design and applications”. *Int. Jour. of Robust and Nonlinear Control*, vol 11, pp.1257-1284, 2001.

[7] T.I. Fossen, M. Blanke, “Nonlinear output feedback control of underwater vehicle propellers using feedback from estimated axial flow velocity”. *IEEE Journal of Oceanic Eng*., vol. 25, No 2, pp 241-255, 2000.

[8] S.S. Hu, P.H. Yang, J.Y Juang and B.C Chang, “Robust nonlinear ship course-keeping control by H_∞ I/O linearization and μ-synthesis”, *Int. Jour. of Robust and Nonlinear Control*, vol 13, pp.55-70, 2003.

[9] J. Ting-Yun Wen, *Control of nonholonomics systems*. Williams S. Levine, editor, The Control Handbook, Florida, USA, CRC Press & IEEE Press, pp. 1359-1368, 1996.

[10] A. Ferrara, L. Giacomini, C. Vechhio, “Control of nonholonomic Systems with uncertainties via second order sliding modes”, *Proc. of the 2006 American Control Conference*, Minneapolis, Minnesota, USA, June 14-16, pp. 5384-5389, 2006.

[11] I. M. Horowitz, *Synthesis of feedback systems*, Academic Press. New York, 1963.

[12] I. M. Horowitz, “Survey of Quantitative Feedback Theory (QFT)”, *Int. Journal of Robust and Non-linear Control*, vol. 11, no 10, pp.: 887-921, 2001.

[13] C. H. Houpis, S.J. Rassmussen, M. García-Sanz, *Quantitative Feedback Theory: fundamentals and applications*, 2nd Edition, CRC Taylor & Francis: Boca Ralen, 2006.

[14] O.Yaniv, *Quantitative Feedback design of linear and nonlinear control systems*, Kluwer Academic Publishers: Massachusetts, 1999.

[15] N. Niksefat, N. Sepehri and Q. Wu, ”Design and experimental evaluation of a QFT contact task controller for electro-hydraulic actuators”, Intern. Journal of Robust and Nonlinear Control, vol. 17, pp. 225-250, 2007.

[16] M. Karpenko and N. Sepehri, “QFT Synthesis of a Position Controller for a Pneumatic Actuator in the presence of Worst-Case persistent disturbances”, *Proc. American Control Conf.*, pp.3158-3163, 2006.

[17] S.F. Wu, M.J. Grimble and S.G. Breslin, “Introduction to Quantitative Feedback Theory for lateral robust flight control systems design”, *Control Engineering Practise*, vol. 6, pp. 805-828, .1998.

[18] I.M. Horowitz and Sidi,”Synthesis of feedback systems with large plant ignorance for prescribed time domain tolerances”, *Int. Journal of Control,* vol. 16(2), pp. 287-309, 1972.

[19] I.M., Horowitz: “Quantitative Synthesis of Uncertain Multi Input-Output Feedback Systems”, *International Journal of Control,* vol. 30, (1), pp. 81-106, 1979.

[20] O. Yaniv and I.M. Horowitz, “A quantitative design method for MIMO linear feedback systems having uncertain plant”, Inter. J. Control, vol. 43, (2), pp. 401-421, 1986.

[21] I.M. Horowitz, “Synthesis of feedback systems with nonlinear time-varying uncertain plants to satisfy quantitative performance specifications”, *Proceedings of the IEEE*, vol.64, pp.123-130, 1976.

[22] C. Borguesani, Y. Chait and O. Yaniv, *Quantitave Feedback Theory Toolbox-for use with Matlab*. The Mathworks Inc. Natick. M.A, 1995.

[23] J. Aranda, J.M Díaz, R. Muñoz- Mansilla, ‘”QFT Control of a coupled system applied to a fast ferry”, *Proc. of 16th IFAC World Congress*. Prague, July 2005.

[24] R. Muñoz-Mansilla, J. Aranda, J.M. Díaz and J.M, Cruz, “A parametric model identification of high-speed craft dynamics”. Ocean Engineering, vol. 6, pp.1025-1038, 2009.

[25] Bazán National Company, *Sea behavior tests of the Turbo Ferry TF-120*. OTI-2086-CM-1, 1995.

[26] A. N. Andry, E.Y. Shapiro and J.C Chung, “Eigenstructure assignment for linear systems”, *IEEE Trans. Aerops. Electron. Syst.*, vol. 19(5), pp.711-729, 1983.

[27] J.M. Cruz, P. Ruipérez, J. Aranda, J, “An Eigenstructure Assignment Approach (2)”. *Robust Flight Control, a design challenge,* edited by J.F. Magni, S. Bennani and J. Terlouw. LNCIS n. 224. Springer, 1997.

[28] S. F. Wu, W. Wei and M.J. Grimble, “Robust MIMO control-system design using eigenstructure assignment and QFT”. *IEE Proc. Control Theory App*., vol. 151(2), pp. 198-209, 2004.

[29] J.W. Choi, H.C. Lee, J. Jim Zhu, “Decoupling and tracking control using eigenstructure assignment for linear time-varying systems”, *Int. Journal of Control*, vol. 74 (5), pp. 453-464, 2001.

[30] K.M. Sobel and E.Y. Shapiro, “Eigenstructure Assignment: a tutorial”, *Proc. American Control Conference*, Boston, pp.456-467, 1985.

[31] J.P. Strand and A.J. Sorensen, “Marine Positioning systems”. *Ocean Engineering Handbook*. (F. El-Hawary, Ed.) Cap. 3, pp. 163-176. CRC Press, USA, 2000.

[32] H. Kajiwara,W. Koterayama, M. Nakamura, and P. Gahinet, “LMI-based design of dynamic positioning control laws for a moored floating platform model”. *Proc. 14th SICE Kyushu Branch Annu. Conf.* pp, 181-184, 1995.

[33] A. Baños, O Yaniv and F. Montoya, “Non-linear QFT synthesis by local linearization”, *Intern. Journal Control,* vol. 79(5), pp. 429-436, 2003.

[34] K. Tanaka, M. Iwasaki and H. Wang, “Switching Control of and R/C Hovercraft. Stabilization and Smooth Switching”, *IEEE Transactions on System, Man, and Cybernetics*, vol. 31(6), pp. 853-863, 2001.

[35] T.I. Fossen, *Guidance and Control of Ocean Vehicles*. Chichester: John Wiley & Sons Ltd. 1994.

[36] A. Baños, “Nonlinear quantitative feedback theory”, *Int. J. of Robust and Nonlinear Control,* vol. 17, pp. 181-202, 2007.

[37] E. Zeidler, *Nonlinear Functional Analysis and its Applications*, Springer-Verlag, New York, 1986.