On the Impossibility of Implementing Perpetual Failure Detectors in Partially Synchronous Systems*

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Abstract. In this paper we study the implementability of different classes of failure detectors in several models of partial synchrony. We show that no failure detector with perpetual accuracy (namely, P, Q, S, and W) can be implemented in any of the models of partial synchrony proposed in [2, 4] in systems with even a single failure. We also show that, in these models of partial synchrony, it is necessary a majority of correct processes to implement a failure detector of class Θ.

1 Introduction

The Consensus problem is considered one of the fundamental problems in distributed computing. However, it was shown by Fischer et al. [5] that the Consensus problem cannot be solved deterministically in an asynchronous system in which processes can fail. This result generated a series of works that tried to identify the amount of synchrony needed to solve Consensus in the presence of failures, and showed how to solve Consensus in these partially synchronous systems [3, 4].

An alternative and elegant approach to circumvent the unsolvability of Consensus in asynchronous systems was proposed by Chandra and Toueg [2]. They augmented the asynchronous model of computation with unreliable failure detectors. Informally, an unreliable failure detector is a distributed "oracle" that gives (possibly incorrect) hints about which processes of the system have crashed. Based on two basic abstract properties (namely, completeness and accuracy), Chandra and Toueg proposed eight different classes of unreliable failure detectors, and showed that Consensus could be solved in an asynchronous system with any of them.

Chandra-Toueg’s model of unreliable failure detectors can be viewed as an abstract way of incorporating partial synchrony assumptions into the model of computation. Instead of focusing on the timing assumptions of a given model of partial synchrony, their model of failure detectors considers abstract properties that must be satisfied in order to solve Consensus. However, the synchrony assumptions are in fact encapsulated in the failure detector. Clearly, systems using these unreliable failure detectors are no longer truly asynchronous; they merely produce the illusion of an asynchronous system by encapsulating all references to time in the failure detector. This leads to the practical problem of implementing a given failure detector in a specific model of synchrony.

From the FLP impossibility result [5] and the possibility of solving Consensus using unreliable failure detectors [2], it can be derived the impossibility of implementing any of Chandra-Toueg’s classes of failure detectors in a purely asynchronous system. (Such an implementation could be used to solve Consensus in an asynchronous system, contradicting the FLP impossibility result.) On the other hand, in a fully synchronous system even a perfect failure detector (i.e., one that does not make mistakes) can be implemented. In such a system, one can build a simple timeout-based algorithm that reliably detects the failure of processes.

1.1 Our Results

In this paper we study the possibility of implementing several classes of failure detectors in partially synchronous systems. We start with the eight classes of failure detectors proposed in [2]. There are already algorithms that implement four of them (P, Q, S, and W) in partially synchronous systems [2, 6]. (We call these classes eventual classes,

because they have eventual accuracy.) That leaves us with only four more classes to consider \((P, Q, S, \text{and } W)\), which we call perpetual classes. We show here that none of these four classes can be implemented in a partially synchronous system with failures (even with one single failure). The partial synchrony assumptions we make in our system are at least as strong as those made in \([2, 4]\), which means that our results apply to their models of partial synchrony as well.

We complete this paper showing that it is impossible to implement a failure detector of class \(\Theta\) in these partially synchronous systems if there is not a majority of correct processes. The class \(\Theta\) of failure detectors was proposed by Aguilera et al. in \([1]\), where it was shown that it is the weakest failure detector that solves uniform reliable broadcast. Since in \([1]\) was also presented an algorithm implementing \(\Theta\) in an asynchronous system with a majority of correct processes, our result identifies a necessary and sufficient condition for \(\Theta\) failure detectors to be implemented in partially synchronous systems.

The rest of the paper is organized as follows. In Section 2 we present the system model we will consider in the paper. In Section 3 we show that the perpetual failure detector classes cannot be implemented in our models of partial synchrony. Finally, in Section 4 we show that it is necessary a majority of correct processes to implement a \(\Theta\) failure detector in our models of partial synchrony.

## 2 System Model

We consider a distributed system consisting of a finite set \(\Pi\) of \(n\) processes, \(\Pi = \{p_1, p_2, \ldots, p_n\}\), that communicate only by sending and receiving messages. Every pair of processes is assumed to be connected by a communication channel.

Processes can fail by crashing, that is, by prematurely halting. Crashes are permanent, i.e., crashed processes do not recover. In every run of the system we identify two complementary subsets of \(\Pi\): the subset of processes that do not fail, denoted correct, and the subset of processes that fail, denoted crashed. We use \(f\) to denote a known upper bound on the number of crashed processes in the system in any run, which we assume is always less than \(n\), i.e., \(|\text{crashed}| \leq f < n\). We also assume that failures are symmetric, i.e., a priori any process in the system is allowed to crash.

### 2.1 Failure Detectors

As we said above, an unreliable failure detector is an oracle that gives hints about crashed processes. In a system with a failure detector, each process has access to a local failure detector module, which monitors other processes in the system and maintains a set of those that it currently suspects to have crashed. A failure detector module can make mistakes by not suspecting a crashed process or by erroneously adding processes to its set of suspects, i.e., it can suspect that a process \(p\) has crashed even though \(p\) is still running. If it later finds that suspecting \(p\) was a mistake, it can remove \(p\) from its set of suspects. Thus, each module may repeatedly add and remove processes from its set of suspected processes. Furthermore, at any given time the failure detector modules at two different processes may have different sets of suspects.

Chandra and Toueg characterized a class of failure detectors by specifying the completeness and accuracy properties that failure detectors in that class must satisfy. Roughly speaking, the completeness property requires that every process that actually crashes is eventually suspected, while the accuracy property restricts the mistakes (i.e., false suspicions) that a failure detector can make. Chandra and Toueg defined two completeness and four accuracy properties in \([2]\), which combined gave rise to eight classes of failure detectors. Regarding completeness, they proposed the following two properties:

- **Strong Completeness**: Eventually every process that crashes is permanently suspected by every correct process.
- **Weak Completeness**: Eventually every process that crashes is permanently suspected by some correct process.

To be useful, a failure detector must also satisfy some accuracy, which restricts the mistakes that it can make. Chandra and Toueg consider the following four accuracy properties:

- **Perpetual Strong Accuracy**: No process is suspected before it crashes.
- **Perpetual Weak Accuracy**: Some correct process is never suspected.
- **Eventual Strong Accuracy**: There is a time after which correct processes are not suspected by any correct process.
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Fig. 1. Eight classes of failure detectors defined in terms of completeness and accuracy.

- **Eventual Weak Accuracy**: There is a time after which some correct process is never suspected by any correct process.

  Note that failure detectors with eventual accuracy may suspect every process at one time or another, while failure detectors with perpetual accuracy require that at least one correct process is never suspected.

  Combining one of the two completeness properties with one of the four accuracy properties we obtain a class of failure detectors. There are eight different classes, which are presented in Figure 1. In this paper we denote the four classes with perpetual accuracy as *perpetual*, and the four classes with eventual accuracy as *eventual*. As we said, Chandra and Toueg showed in [2] that any of the failure detectors of Figure 1 can be used to solve Consensus.

  We now define the class $\Theta$ of failure detectors [1] in terms of completeness and accuracy properties. A failure detector of class $\Theta$ must satisfy the following properties. (We say that a process $p$ trusts another process $q$ at a given time $t$ if $p$ does not suspect $q$ at time $t$.)

- $\Theta$-completeness. There is a time after which correct processes do not trust any process that crashes.

- $\Theta$-accuracy. If there is a correct process then, at every time, every process trusts at least one correct process.

  Note that a process may be trusted even if it has actually crashed. Moreover, the correct process trusted by a process $p$ is allowed to change over time (in fact, it can change infinitely often), and it is not necessarily the same as the correct process trusted by another process $q$.

  As we said, $\Theta$ is the weakest class of failure detectors that solves *uniform reliable broadcast*, and Aguilera et al. proposed in [1] an algorithm implementing it in an asynchronous system with a majority of correct processes.

2.2 Partial Synchrony

In order to define the level of synchrony of a system we use two parameters, the transmission delay of messages and the relative speeds of processes. In the asynchronous model there are no upper bounds on one or both of these parameters. Thus, to say that a system is asynchronous is to make no timing assumptions. In the synchronous model there are known upper bounds, which we denote by $\Delta$ and $\Phi$, respectively, on the transmission delay of messages and the relative speeds of processes. From the synchronous to the asynchronous models there is a whole range of possible models of synchrony. We call these *partially synchronous models*.

Dwork et al. [4] consider the following two models of partial synchrony:

- $M_1$: in every run of the system, there are upper bounds $\Delta$ and $\Phi$ on the transmission delay of messages and the relative speeds of processes, respectively, but these bounds are not known.

- $M_2$: bounds exist and are known, but they hold only after some unknown (but finite) time $GST$ (for *Global Stabilization Time*). Messages sent before $GST$ can get lost.

A system that conforms to model $M_2$ can be seen as asynchronous up to $GST$, and as synchronous after $GST$. Thus, $M_2$ can be seen as an *eventually synchronous model*. However, it is important to note that the actual value of $GST$ is not known and can vary from run to run.

Dwork et al. [4] showed that Consensus can be solved in both models $M_1$ and $M_2$ with a majority of correct processes, i.e., when $f < n/2$. They proposed Consensus algorithms for various fault models that work correctly

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1 $\Theta$-completeness is the same as strong completeness, since a trust is just the complement of a suspicion.

2 Model $M_2$ becomes interesting if channels are unreliable before $GST$. In such a case, algorithms must include techniques to mask the loss of messages.
regardless of the actual values of the bounds (in the case of model $M_1$), and the actual value of $GST$ (in the case of model $M_2$).

In [2], Chandra and Toueg proposed a weaker model of partial synchrony $M_3$, which generalizes the two previous models $M_1$ and $M_2$:

- $M_3$: bounds $\Delta$ and $\Phi$ exist, but they are not known and they hold only after some unknown (but finite) time $GST$.

A system that conforms to model $M_3$ can be seen as asynchronous up to $GST$, and as a system conforming to model $M_1$ after $GST$. Note also that every system that conforms to models $M_1$ or $M_2$ also conforms to model $M_3$.

Chandra and Toueg showed how to implement a failure detector of class $\Diamond \mathcal{P}$ in a system that conforms to model $M_3$. This shows that the four classes of eventual failure detectors can be implemented in such a system model, since a $\Diamond \mathcal{P}$ failure detector also belongs to classes $\Diamond \mathcal{W}$, $\Diamond \mathcal{S}$, and $\Diamond \mathcal{Q}$. Concerning the perpetual classes of failure detectors, i.e., $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{W}$, they were neither shown to be implementable nor impossible to implement in models of partial synchrony.

In this paper, we prove the impossibility of implementing such perpetual classes of failure detectors in partially synchronous models of computation. When proving impossibility results, it is convenient to consider the strongest model of partial synchrony, because the impossibility applies directly to the weaker ones. Hence, we will consider in our proofs of impossibility models $M_1$ and $M_2$. Furthermore, when considering model $M_1$ we will assume that the bound on the relative speeds of processes $\Phi$ is known, while only the bound on the transmission delay of messages $\Delta$ is unknown\(^3\). Clearly, this model is stronger than $M_1$ and $M_3$, and any negative result will apply to these models as well. We will also assume that communication channels are completely reliable under both models. As we will see, the impossibility proofs are the same for both models, with minor variations, which will be pointed out.

2.3 Any Implementation of a Perpetual Failure Detector in $M_1$ Requires a Majority of Correct Processes

There is a simple proof that any implementation in $M_1$ of a perpetual failure detector requires a majority of correct processes. The proof basically shows that any implementation of a failure detector of class $\mathcal{W}$ in the model of partial synchrony $M_1$ (and thus in $M_3$) requires $f < n/2$.

The proof, which uses contradiction, goes as follows. It is shown in [4] that the smallest number of processes for which an $r$-resilient Consensus protocol exists in the model of partial synchrony $M_1$ is $2r + 1$. In other words, any protocol that solves Consensus in model $M_1$ requires a majority of correct processes.

Let us assume now that we have an algorithm $A$ that implements a failure detector of class $\mathcal{W}$ in model $M_1$ with $f \geq n/2$. In [2], Chandra and Toueg propose a Consensus protocol based on $\mathcal{W}$ that tolerates up to $n-1$ faulty processes in asynchronous systems with $n$ processes. In other words, Chandra-Toueg's protocol does not require a majority of correct processes. Clearly, one could run this protocol on top of $A$ and solve Consensus in model $M_1$ with $f \geq n/2$, which is a contradiction.

Note that this argument shows that $\mathcal{W}$ cannot be implemented in the model of partial synchrony $M_1$ without a majority of correct processes, but it says nothing about the possibility of implementing $\mathcal{W}$ with a majority of correct processes, i.e., when $f < n/2$. In the following section we show the impossibility even when there is only one faulty process. Furthermore, the above proof only applies to the models $M_1$ and $M_3$ of partial synchrony, while the results of the following section also apply to model $M_2$.

3 Impossibility of Implementing Perpetual Failure Detectors

In this section, we show that none of the perpetual failure detector classes ($\mathcal{P}$, $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{W}$) can be implemented in our models of partial synchrony. From the relationship between failure detector classes described in [2], it is sufficient to show the impossibility result for the failure detector class $\mathcal{W}$, since $\mathcal{W}$ is the weakest of the four classes of failure detectors satisfying perpetual accuracy. For simplicity, we show the result for the failure detector class $\mathcal{S}$, and the result applies directly to $\mathcal{W}$ since both classes are equivalent [2]. The approach followed is assuming the existence of

\(^{3}\) Actually, the results hold if at least one of the two bounds is unknown. Intuitively, any slowness of the relative speeds of processes can always be attributed to the slowness of the transmission delay of messages and vice versa.

\(^{4}\) Actually, their protocol is based on $\mathcal{S}$, but they also show that failure detector classes $\mathcal{W}$ and $\mathcal{S}$ are equivalent.
a failure detector satisfying the strong completeness property, and showing that the perpetual weak accuracy property is violated.

The principle used to prove the impossibility is to consider different runs of the system — with and without crashes — such that they look identical for some correct processes up to certain time \( t \). Hence, these processes can take the same actions in both kinds of runs up to \( t \), in particular in what concerns the suspicion of other processes. We show that by doing this, perpetual weak accuracy is violated, and thus the failure detector does not implement any of the four perpetual failure detector classes defined in [2]. To construct a run without a crash that looks identical up to time \( t \) to one with a crash, we assume that the appropriate messages are delayed beyond \( t \). This can happen if the value of the parameter \( \Delta \) or \( GST \) (depending on the synchrony model) is larger than \( t \). This is a valid assumption, since the values of these parameters are unknown, and can be chosen freely for a given run if required.

Following the previous approach, we show an admissible run of the system in which all the correct processes are erroneously suspected at least once, thus violating perpetual weak accuracy. Let \( \Sigma \) be a partially synchronous distributed system that conforms to model \( M_1 \) or model \( M_2 \), made up of \( n > 1 \) processes, such that at least one of them is correct, i.e., at most \( f < n \) of them may crash.

**Theorem 1.** Let \( FD_\Sigma \) be a failure detector, implemented on the system \( \Sigma \), that satisfies the strong completeness property. Then \( FD_\Sigma \) cannot satisfy the weak accuracy property.

**Proof.** Let us consider a run \( R_1 \) of \( \Sigma \) in which only process \( p_1 \) crashes, doing it at time \( t_0 = 0 \). Since \( FD_\Sigma \) satisfies the strong completeness property, there is some time \( t_1 \) at which all other processes permanently suspect \( p_1 \) in \( R_1 \).

Let us consider now a run \( R_2 \) of \( \Sigma \) in which only process \( p_2 \) crashes, doing it at the time \( t_1 \) defined in \( R_1 \), and all messages sent by \( p_1 \) are received after \( t_1 \) (this can happen if we assume that \( \Delta > t_1 \), if \( \Sigma \) conforms to \( M_1 \), or \( GST > t_1 \), if \( \Sigma \) conforms to \( M_2 \)). Also in \( R_2 \), all processes except \( p_1 \) behave exactly like in run \( R_1 \) up to time \( t_1 \). Clearly, all correct processes, except \( p_1 \), cannot distinguish run \( R_2 \) from run \( R_1 \) up to time \( t_1 \). Hence, at time \( t_1 \) they will suspect \( p_1 \) in \( R_2 \), as they did in \( R_1 \). Finally, since \( FD_\Sigma \) satisfies the strong completeness property, there is some time \( t_2 \geq t_1 \) at which all other processes permanently suspect \( p_2 \) in \( R_2 \).

Generalizing this reasoning, we obtain \( n \) runs \( R_i \), \( i = 1, \ldots, n \) of \( \Sigma \) as follows. In run \( R_i \) only process \( p_i \) crashes, doing it at time \( t_i-1 \), defined in \( R_{i-1} \), and for each process \( p_k \), \( k = 1, \ldots, i-1 \), all messages sent by \( p_k \) after \( t_{k-1} \) are received after \( t_k \), with \( t_0 = 0 \) (this can happen if we assume that \( \Delta > t_{i-1} \), if \( \Sigma \) conforms to \( M_1 \), or \( GST > t_{i-1} \), if \( \Sigma \) conforms to \( M_2 \)). Also in \( R_i \), for each process \( p_k \), \( k = 1, \ldots, i-1 \), all processes except \( p_k \) behave exactly like in run \( R_k \) up to time \( t_k \). Clearly, for each process \( p_k \), \( k = 1, \ldots, i-1 \), all correct processes except \( p_k \) cannot distinguish run \( R_i \) from run \( R_k \) up to time \( t_k \). Hence, at time \( t_k \) they will suspect \( p_k \) in \( R_i \), as they did in \( R_k \). Finally, since \( FD_\Sigma \) satisfies the strong completeness property, there is some time \( t_i \geq t_{i-1} \) at which all other processes permanently suspect \( p_i \) in \( R_i \).

Let us now consider a run \( R \) of \( \Sigma \) in which no process crashes, but:

- All messages sent by \( p_i \) after time \( t_{i-1} \) as defined in \( R_{i-1} \) are received after time \( t_i \) as defined in \( R_i \). This can happen if we assume that \( \Delta > t_{i-1} \), if \( \Sigma \) conforms to \( M_1 \), or \( GST > t_{i-1} \), if \( \Sigma \) conforms to \( M_2 \).
- For each process \( p_i \), \( i = 1, \ldots, n \), all processes except \( p_i \) behave exactly like in run \( R_i \) up to time \( t_i \).

Clearly, for each process \( p_i \), \( i = 1, \ldots, n \), all processes except \( p_i \) cannot distinguish run \( R \) from run \( R_i \) up to time \( t_i \). Hence, at time \( t_i \) they will suspect \( p_i \) in \( R \), as they did in \( R_i \). Since this is true for every process \( p_i \), \( i = 1, \ldots, n \) in the system, in run \( R \) all correct processes are suspected at some time by the rest of correct processes, and the weak accuracy property is not satisfied.

**Corollary 1.** There is no protocol that implements a failure detector of class \( S \) in a partially synchronous distributed system that conforms to model \( M_1 \) or model \( M_2 \).

**Corollary 2.** There is no protocol that implements a failure detector of class \( W \) in a partially synchronous distributed system that conforms to model \( M_1 \) or model \( M_2 \).

**Proof.** Follows from Corollary 1 and the fact that \( S \) and \( W \) are equivalent [2].

\(^{5}\) Note that an algorithm implementing any given class \( D \) of failure detectors must satisfy the properties that characterize \( D \) in all admissible runs.
Corollary 3. There is no protocol that implements a failure detector of classes $\mathcal{P}$ or $\mathcal{Q}$ in a partially synchronous distributed system that conforms to model $M_1$ or model $M_2$.

Proof. Follows from Corollary 2 and the fact that $\mathcal{P}$ and $\mathcal{Q}$ are stronger than $\mathcal{W}$ [2].

4 Impossibility of Implementing $\Theta$ without a Majority of Correct Processes

In this section, we show that the failure detector $\Theta$, proposed by Aguilera et al. in [1], cannot be implemented in the models of partial synchrony $M_1$ and $M_2$ without a majority of correct processes.

Theorem 2. Let $\Sigma$ be a partially synchronous distributed system that conforms to model $M_1$ or model $M_2$, made up of $n > 1$ processes. Let $FD_{\Sigma}$ be a failure detector, implemented on the system $\Sigma$, that satisfies the $\Theta$-completeness property. Then, if $f \geq \lceil n/2 \rceil$, $FD_{\Sigma}$ cannot satisfy the $\Theta$-accuracy property.

Proof. Let us consider a run $R$ of $\Sigma$ in which processes $p_1, p_2, \ldots, p_{\lceil n/2 \rceil}$ crash at time 0. Since $FD_{\Sigma}$ satisfies the $\Theta$-completeness property, there is some time $t$ at which $p_{n}$ permanently suspects all these processes.

Let us now consider a run $R'$ in which processes $p_{\lceil n/2 \rceil+1}, \ldots, p_{n}$ crash at time $t+1$, and:

- All messages sent by processes $p_1, p_2, \ldots, p_{\lceil n/2 \rceil}$ are received after time $t$. This can happen if we assume that $\Delta > t$, if $\Sigma$ conforms to $M_1$, or GST $> t$, if $\Sigma$ conforms to $M_2$.
- Each process $p_{\lceil n/2 \rceil+1}, \ldots, p_{n}$ behaves exactly like in run $R$ up to time $t$.

Clearly, process $p_{n}$ cannot distinguish run $R'$ from run $R$ up to time $t$. Hence, at time $t$ it will suspect all the processes $p_1, p_2, \ldots, p_{\lceil n/2 \rceil}$ which are the correct processes of the run $R'$. Hence, in run $R'$ all correct processes are suspected at time $t$ by process $p_{n}$, and the $\Theta$-accuracy property is not satisfied, since there is a time at which some process does not trust any correct process.

Corollary 4. There is no protocol that implements a failure detector of class $\Theta$ in a partially synchronous distributed system that conforms to model $M_1$ or model $M_2$ without a majority of correct processes.

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