Localization in unknown environments using computer vision

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Introduction

- Computer vision
  - Cameras
  - Robots
  - Localization
- Kalman Filter

Computer Vision

- Advantages
  - More information
  - 3D objects, colors, shapes
- Disadvantages
  - High execution time
  - Poor reliability
  - No depth

Example - Faces

037	084	132	155	172	175	178	179	181	180	178	177	176	173	168	163	158	156	155	149	139	128	117	111	096	087	069	052	049	040	021	021
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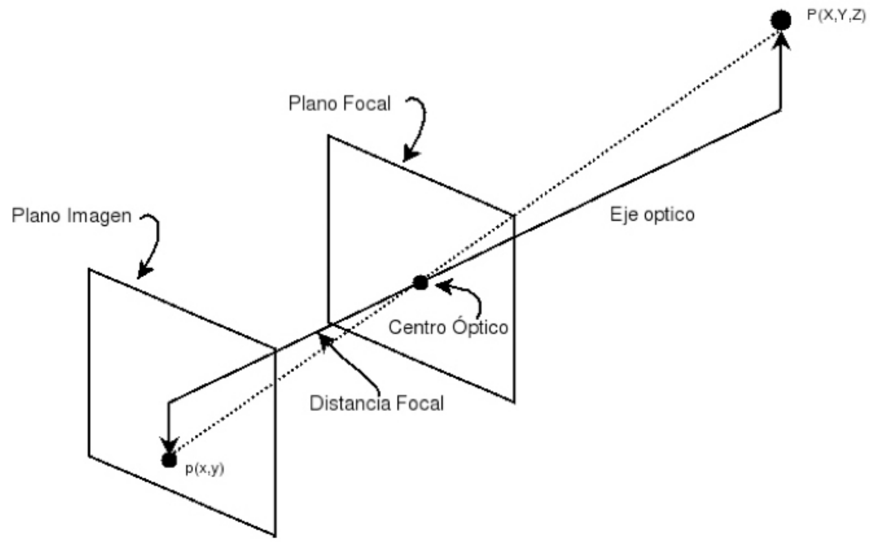
Pin-Hole Cameras

- Intrinsic parameters
  - Focal distance
  - Optical center
  - Skew
  - Others
- Extrinsic parameters
  - 3D position
  - Focus of attention
  - Roll



Quaternions

- Extension of complex numbers
- Represents a 3D rotation with 4 elements:
  - q_0: Real scalar, rotation angle
  - $q1, q2, q3$: Imaginary vector, rotation axis
- Easy translation to rotation matrices
- Rotation composition with Hamilton product

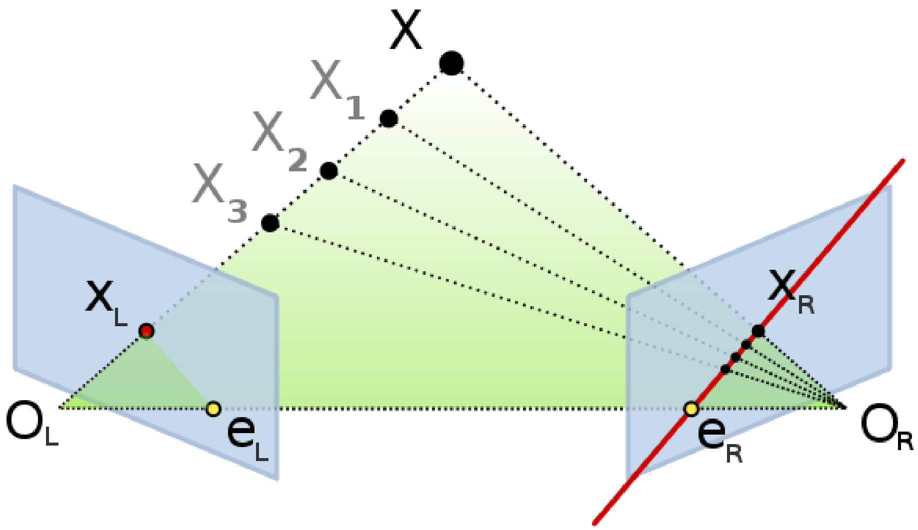
Project and unproject

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} + \begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix} \begin{bmatrix} -\frac{x}{z} \\ -\frac{y}{z} \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{u - u_0}{f_x} \\ \frac{v - v_0}{f_y} \\ 1{,}0 \end{bmatrix}$$

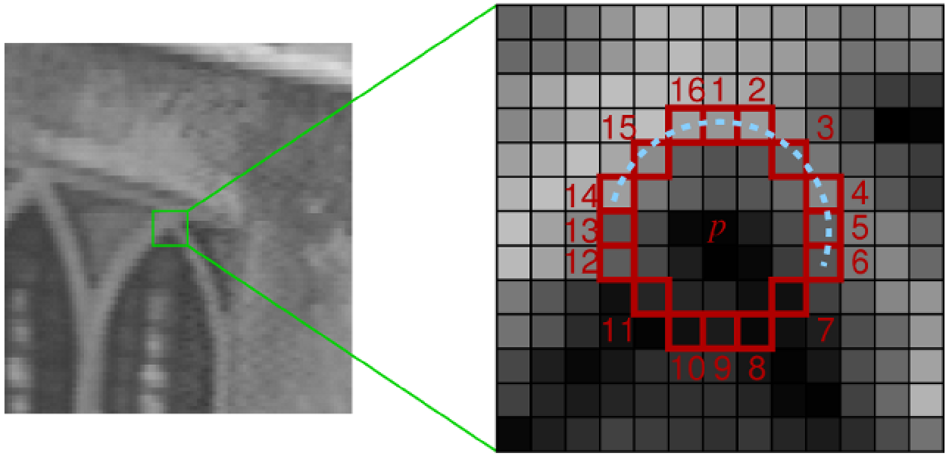
Getting 3D

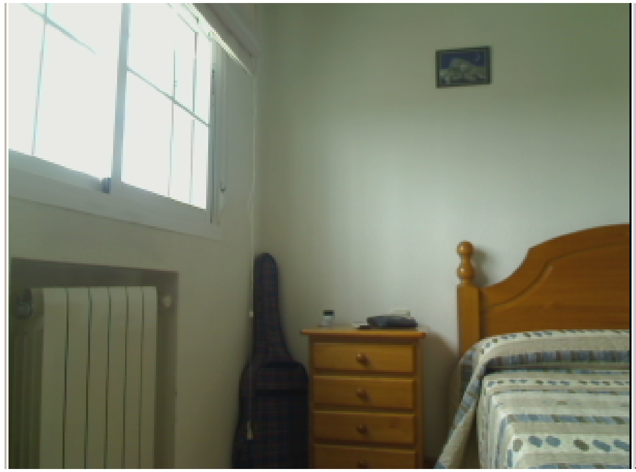
- Epipolar geometry:
  - Two cameras at the same time
  - One camera at different times (and movement)

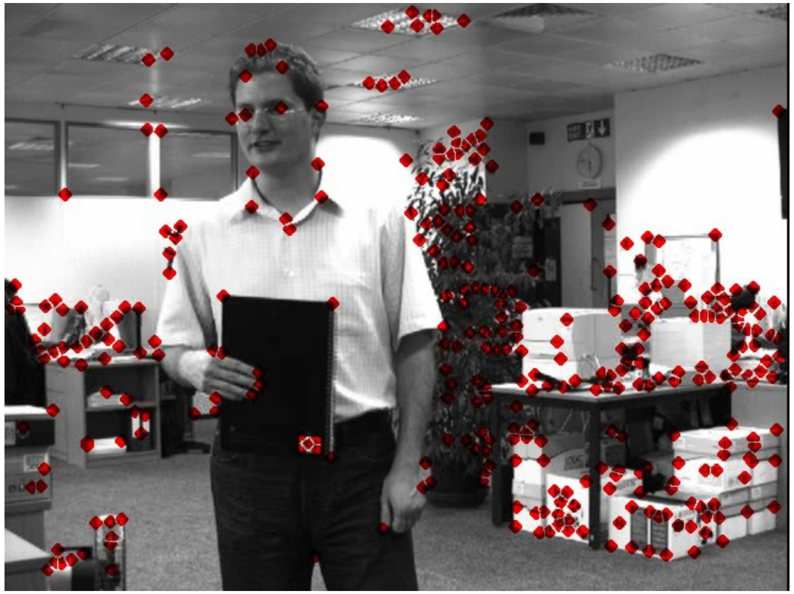


Features detection

- Feature types:
  - Points
  - Lines
  - Complex objects
- Techniques:
  - Color filters
  - Edge filters (Canny, Sobel)
  - Morphology
  - Hough transform
  - Shi-Tomasi
  - FAST Corner Detection

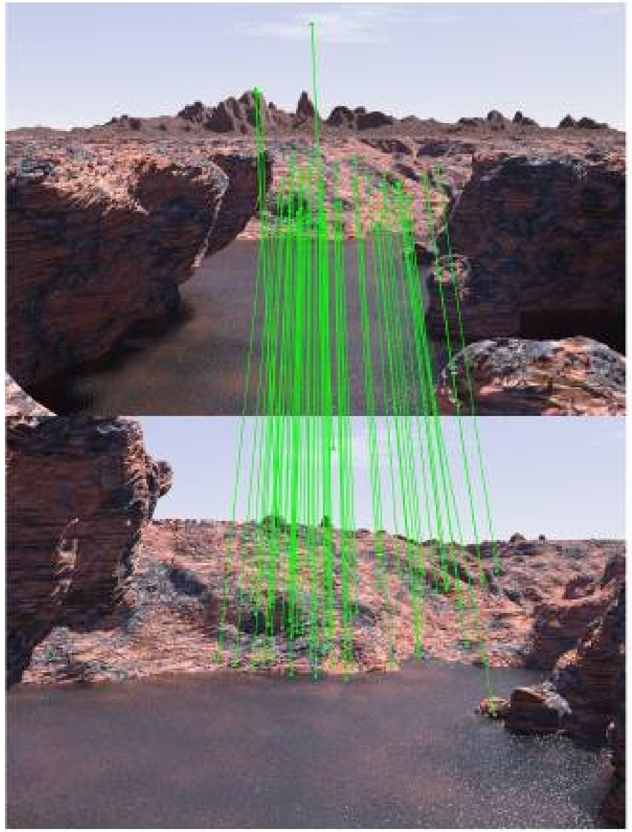






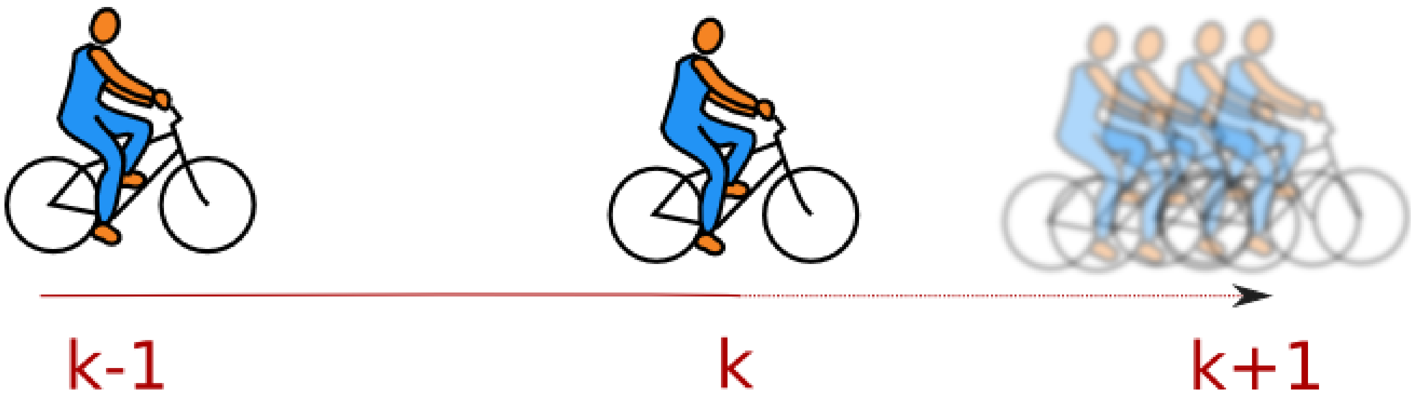
Feature matching

- With descriptors:
  - SURF
  - SIFT
  - ORB
- Patch to Patch:
  - Difference
  - Normalized squared difference
  - Avg color, color counting, ...



Kalman Filter

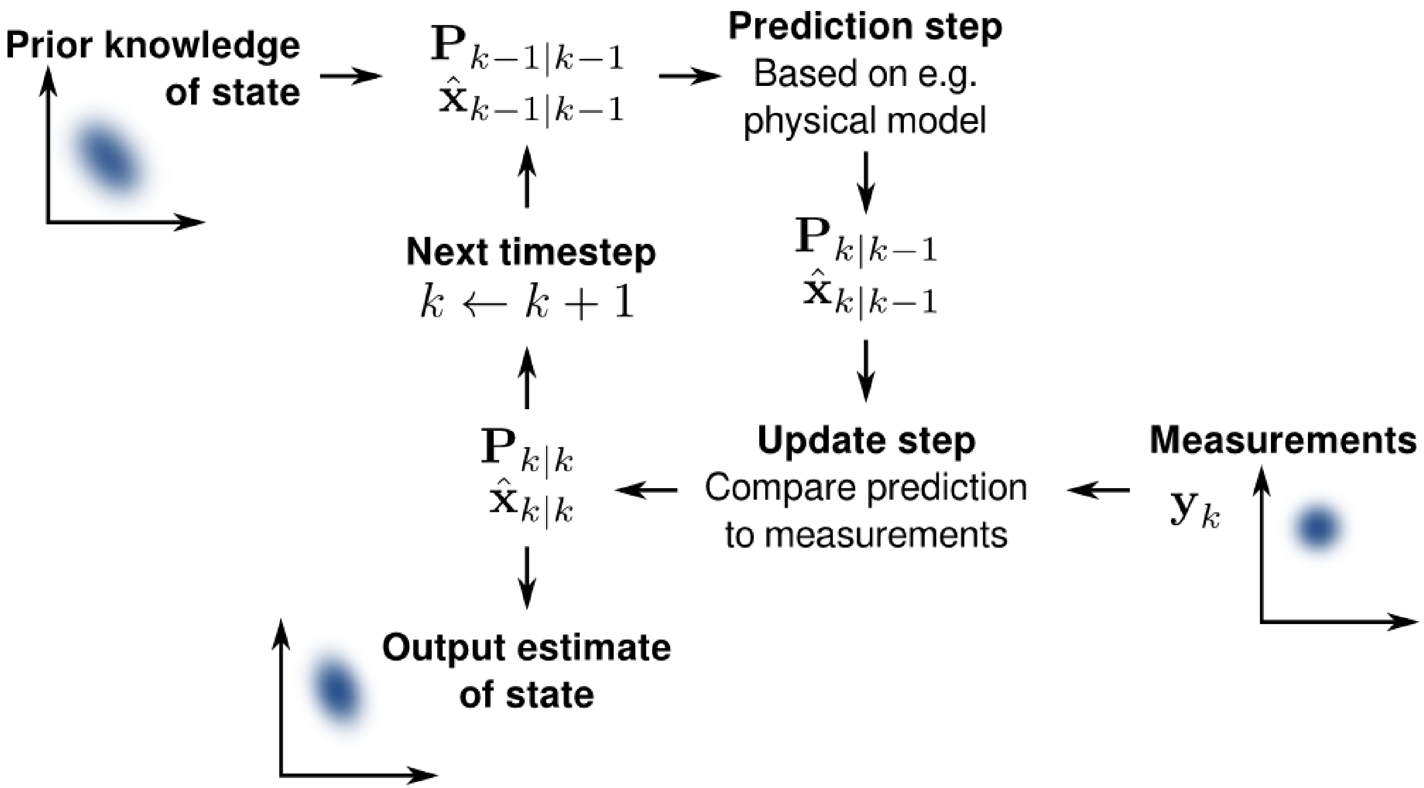
- Linear systems and Gaussian distributions
- Estimates state based on observations



Kalman Properties

- Main Matrices:
  - $\hat{x}_k$: Estimated status vector
  - P_k: Estimated covariance matrix
  - y_k: Observation vector
- Models:
  - F_k: Prediction model
  - H_k: Observation model
- Steps:
  - Prediction step
  - Update step

Basic algorithm



Complete KF algorithm

Predict

Status prediction	$\hat{x}_{k\|k-1} = F_k \hat{x}_{k-1}$
Covariance prediction	$P_{k\|k-1} = F_k P_{k-1} F_k^T + Q_k$

Update

Residual	$\tilde{y}_k = z_k - H_k \hat{x}_{k\|k-1}$
Residual covariance	$S_k = H_k P_{k\|k-1} H_k^T + R_k$
Kalman Gain	$K_k = P_{k\|k-1} H_k^T S_k^{-1}$
Status stimation	$\hat{x}_k = \hat{x}_{k\|k-1} + K_k + \tilde{y}_k$
Covariance stimation	$P_k = (I - K_t H_k) P_{k\|k-1}$

Extended Kalman Filter

- Use Kalman with non linear systems
- More dynamic systems but less optimal
- Linearize models:
  - $F_k \rightarrow J_{F_k}$ and $f()$
  - $H_k \rightarrow J_{H_k}$ and $h()$

Complete EKF algorithm

Predict	
Status prediction	$\hat{x}_{k\|k-1} = f(\hat{x}_{k-1})$
Covariance prediction	$P_{k\|k-1} = J_{F_k} P_{k-1} J_{F_k}^T + Q_k$
Update	
Residual	$\tilde{y}_k = z_k - h(\hat{x}_{k\|k-1})$
Residual covariance	$S_k = J_{H_k} P_{k\|k-1} J_{H_k}^T + R_k$
Kalman Gain	$K_k = P_{k\|k-1} J_{H_k}^T S_k^{-1}$
Status stimation	$\hat{x}_k = \hat{x}_{k\|k-1} + K_k + \tilde{y}_k$
Covariance stimation	$P_k = (I - K_t J_{H_k}) P_{k\|k-1}$

Discrete bicycle example

$$x_k = \begin{bmatrix} x \\ v \end{bmatrix} \qquad x_k = Fx_{k-1} + Ga_k$$

$$z_k = Hx_{k-1} + u_k$$

$$F = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$x_k = Fx_{k-1} + w_k$$

$$R = E\begin{bmatrix} u_k & u_k^T \end{bmatrix} = \begin{bmatrix} \sigma_z^2 \end{bmatrix}$$

$$Q = GG^T\sigma_a^2 = \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 \end{bmatrix} \sigma_a^2$$

Continuous bicycle example (Kalman-Bucy filter)

$$X = \begin{bmatrix} x \\ v \end{bmatrix}$$

$$\frac{dx}{dt} = v \qquad \frac{dv}{dt} = a + w$$

$$z = x + u$$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a + \begin{bmatrix} 0 \\ w \end{bmatrix}$$

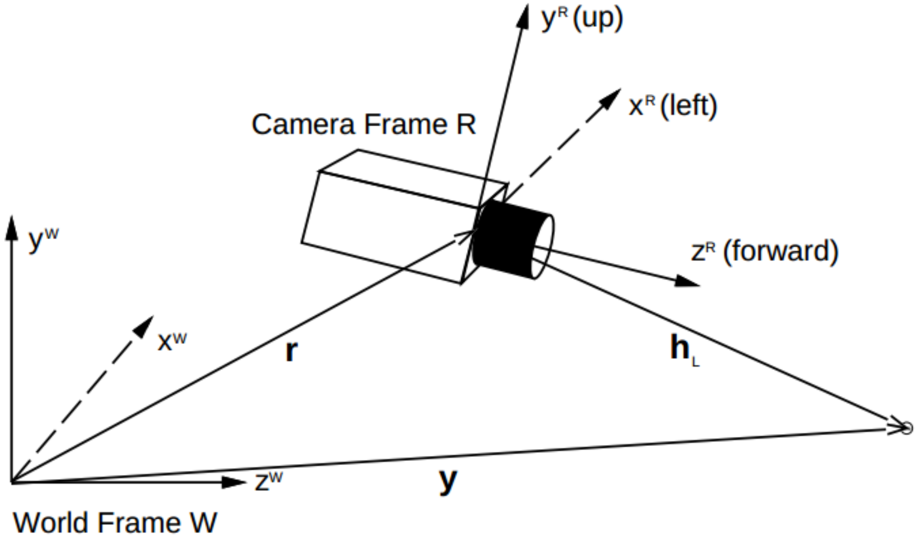
$$z = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + u$$

$$z = HX + u$$

$$\frac{d}{dt}X = FX + Ba + \begin{bmatrix} 0 \\ w \end{bmatrix}$$

MonoSLAM

- Monocular Simultaneous Localization and Mapping
- Calculates camera position and orientation (6 degrees of freedom)
- Creates a map of features automatically
- Uses an EKF to estimate its status



EKF with MonoSLAM

$$\hat{x} = \begin{bmatrix} x_v \\ f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} \qquad P = \begin{bmatrix} P_{x_v x_v} & P_{x_v f_1} & P_{x_v f_2} & \dots & P_{x_v f_N} \\ P_{f_1 x_v} & P_{f_1 f_1} & P_{f_1 f_2} & \dots & P_{f_1 f_N} \\ P_{f_2 x_v} & P_{f_2 f_1} & P_{f_2 f_2} & \dots & P_{f_2 f_N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{f_N x_v} & P_{f_N f_1} & P_{f_N f_2} & \dots & P_{f_N f_N} \end{bmatrix}$$

Status Model

$$x_v = \begin{bmatrix} r^W \\ v^W \\ q^{WR} \\ \omega^R \\ p^W_{f_1} \\ \dots \\ p^W_{f_N} \end{bmatrix} \qquad x_{v|k} = f(x_{v|k-1}) = \begin{bmatrix} r^W_{|k-1} + v^W_{|k-1}\Delta k \\ v^W_{|k-1} \\ q_g(\omega^R_{|k-1}\Delta k) \times q^{WR}_{|k-1} \\ \omega^R_{|k-1} \\ p^W_{f_1|k-1} \\ \dots \\ p^W_{f_N|k-1} \end{bmatrix}$$

Observation Model

- Project each point 3D to camera pixels

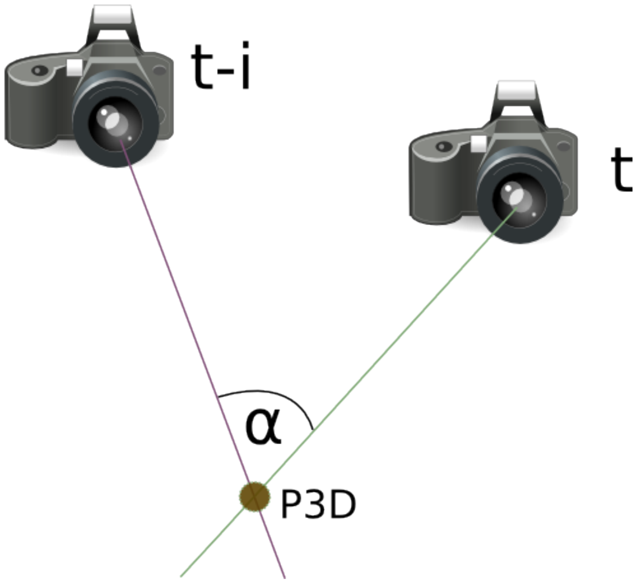
$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = h \begin{bmatrix} x_v \\ f_i^W \end{bmatrix} = K\left(R\left(T\begin{bmatrix} x_v \\ f_i^W \end{bmatrix}\right)\right)$$

Feature Matching

- Features (points) detected with FAST
- Matching patch to patch:
  - Search inside uncertainty regions
  - Normalized squared difference
- Initial features known

Creating new Features

- Initialize on the infinite
- Calculate 3D Matrix:
  - $M = p1 * p2' - p2 * p1'$
- Concatenate matrices:
  - $TM = [TM; M]$
- Get 3D point (autovector) with SVD:



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